W07 Homework B

Due date: Tuesday 3/10, 11:59pm

01 ★

PMF calculations from a table

Suppose the joint PMF of X and Y has values given in this table:

$X \backslash Y$	**0**	**1**	**2**	**3**
1	0.10	0.15	0	0.05
2	0.20	0.05	0.05	0.20
3	0.05	0	x	0.05

(a) Find x.

(b) Find the marginal PMF of X.

(c) Find the PMF of the random variable $Z = XY$.

(d) Find $P[X = Y]$ and $P[X > Y]$.

02 ★

Marginals and probability from joint PDF

Suppose X and Y have joint PDF given by:

$$f_{X,Y}(x,y) = \begin{cases} 2e^{-(x+2y)} & \text{if } x,y>0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the marginal PDFs for X and Y.

(b) Find $P[X>Y]$.

03

Air pollution

In a certain community, levels of air pollution may exceed federal standards for ozone or for particulate matter on some days. In a particular summer week, let X be the number of days on which the ozone standard is exceeded, and let Y be the number of days on which the particulate matter is exceeded.

The following table represents the joint PMF for X and Y.

	$Y = 1$	$Y = 2$	$Y = 3$
$X = 1$	0.09	0.11	0.05
$X = 2$	0.17	0.23	0.08
$X = 3$	0.06	0.15	0.06

(a) Find $P[X > Y]$.

(b) Find $E[X + Y]$.

04 ★

Composite PDF from joint PDF

The joint density of random variables X and Y is given by:

$$f_{X,Y}(x,y) = \begin{cases} e^{-x-y} & x,y>0 \\ 0 & \text{otherwise} \end{cases}$$

Compute the PDF of $Z = X/Y$.

05

One car outlasts the other

Suppose that $X, Y \sim \text{Exp}(0.1)$ are two independent exponential random variables.

(a) Find the joint PDF $f_{X,Y}$.

(b) Find the probability:

$$P\big[\, X - 5 > Y - 15 \mid X > 5 \text{ AND } Y > 15 \,\big]$$

Compare to W05-B Q02 “Vehicle Lifetimes.” Which method is easier? (For yourself, not this HW.)

06

Normal distribution - cars passing toll booth

The number of cars passing a toll booth on Wednesdays has a normal distribution $\mathcal{N}(1200, 40000)$.

(a) What is the probability that on a randomly chosen Wednesday, more than 1,400 cars pass the toll booth?

(b) What is the probability that between 1,000 and 1,400 cars pass the toll booth on a random Wednesday?

(c) Suppose it is learned that at least 1200 cars passed the toll booth last Wednesday. What is the probability that at least 1300 cars passed the toll booth that day?